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Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Wednesday 6 November 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. A curve C has equation $y = 2x^2(x - 5)$

$\frac{dy}{dx} = 0$

(a) Find, using calculus, the x coordinates of the stationary points of C.

(4)

(b) Hence find the values of x for which y is increasing.

(2)

(a) $y = 2x^2(x-5)$

$y = 2x^3 - 10x^2$

DIFFERENTIATION

$$\boxed{\begin{aligned} y &= ax^n \\ \frac{dy}{dx} &= nax^{n-1} \end{aligned}}$$

$\frac{dy}{dx} = (3)2x^{3-1} + (2)(-10x^{2-1})$

$= 6x^2 - 20x$

At stationary point:

$6x^2 - 20x = 0$

$x(6x - 20) = 0$

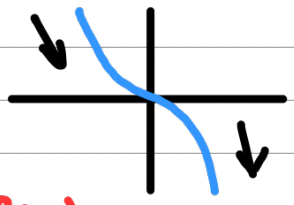
$x = 0$ or $x = \frac{20}{6} = \frac{10}{3}$

(b) where y is increasing, there is a positive gradient.

$[y = 2x^3 - 10x^2]$ when we sketch a graph, we know:

① 

$y = +ax^3$



$y = -ax^3$

our $f(x)$ looks like this!

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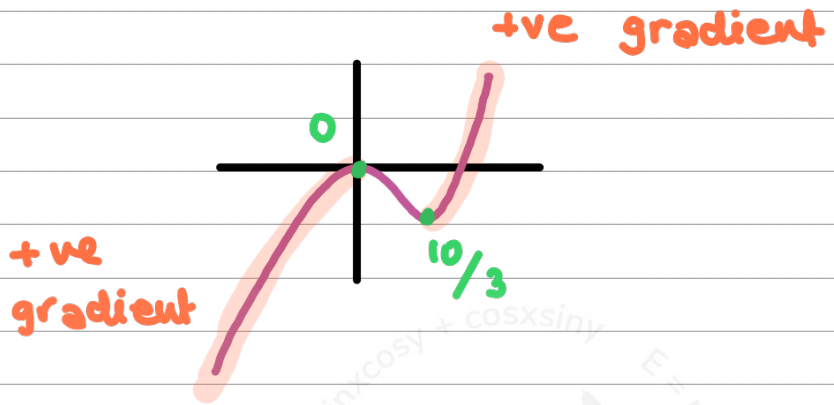
Question 1 continued

② we know our two stationary points where gradient = 0 :



So y is increasing at :

$x \leq 0$, $x \geq 10/3$

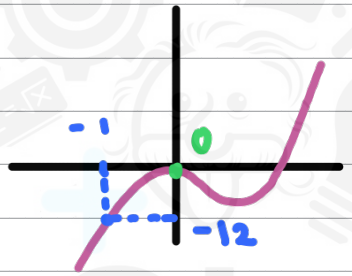
OR : If you want to check further :

① at $x = 0$, $y = 0$

at $x \leq 0$, ex : $x = -1$:

$y = 2(-1)^2(-1-5) = -12$

$y = 0 > y = -12$



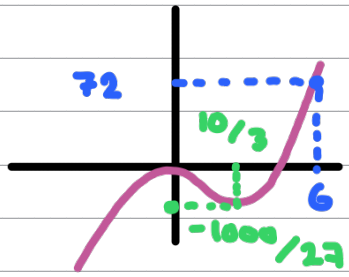
So, y is increasing before $x = 0 \rightarrow x \leq 0$

② at $x = 10/3$, $y = -1000/27$

at $x \geq 10/3$, ex : $x = 6$:

$y = 2(6)^2(6-5) = 72$

$y = -1000/27 < 72$



So, y is increasing after $x = 10/3 \rightarrow x \geq 10/3$

(Total 6 marks)

Q1

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if population in year 1 is x, population in year 2 is 102% of year 1

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2. The adult population of a town at the start of 2019 is 25 000 (a_1)

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

partial sum

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

(a) For this sequence :

$\times \frac{102}{100} = r$

term : 25 000 , ? , ..., x

term order : a_1 , a_2 , a_{14}

year : 2019 , 2020 , ..., 2032

To find the nth term for a geometric sequence :

$a_n = ar^{n-1}$

Where :

n = no. of years INCLUDING 2019 :

$2032 - 2019 = 13$, $13 + 1 = 14$

$r = \frac{102}{100} = 1.02$

a = term 1 (a_1) = 25 000

$a_{14} = (25\,000)(1.02)^{14-1}$

$= 32\,340.16...$

$= 32\,300$ (3 sf)

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Question 2 continued

(b) Use partial sum formula :

$$S_n = \frac{a_1(1-r^n)}{(1-r)}$$

each gives £5

$$S_n = \frac{(25\,000 \times 5)(1-1.02^{14})}{1-1.02}$$

$$= \frac{125\,000\,(1-1.02^{14})}{1-1.02}$$

$$= £1\,996\,742$$

$$= 1\,997\,000 \quad \text{(nearest thousand)}$$

NOTE: For geometric sequence :

$$S_n = \frac{a_1(1-r^n)}{(1-r)} \quad \text{OR} \quad S_n = \frac{a_1(r^n-1)}{(r-1)}$$

both give same answer !

Q2

(Total 6 marks)

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3. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$\left(1+\frac{x}{4}\right)^{12}$$

giving each coefficient in its simplest form.

(3)

has no x

(b) Find the term independent of x in the expansion of

$$\left(\frac{x^2+8}{x^5}\right)\left(1+\frac{x}{4}\right)^{12}$$

(3)

(a) Binomial expansion formula :

$$\boxed{(a+bx)^n = {}^nC_0 a^n (bx)^0 + {}^nC_1 a^{n-1}(bx)^1 + {}^nC_2 a^{n-2}(bx)^2 + \dots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial

r : (position of term) − 1

OR : USE CHOOSE FUNCTION ON CALCULATOR

for $\left(1+\frac{1}{4}x\right)^{12}$:

$$= {}^{12}C_0\, 1^{12}\left(\frac{x}{4}\right)^0 + {}^{12}C_1\, 1^{12-1}\left(\frac{x}{4}\right)^1 + {}^{12}C_2\, 1^{12-2}\left(\frac{x}{4}\right)^2 + {}^{12}C_3\, 1^{12-3}\left(\frac{x}{4}\right)^3$$

(NOTES :- the first and last ${}^nC_r$ are always $=1$
- ${}^nC_1$ is always equal to n)

${}^{12}C_0 = 1$ ${}^{12}C_2 = \frac{12!}{(12-2)!\,2!} = 66$

${}^{12}C_1 = 12$ ${}^{12}C_3 = \frac{12!}{(12-3)!\,3!} = 220$

$$= 1(1)(1) + 12(1)\left(\frac{x}{4}\right) + 66(1)\left(\frac{x^2}{16}\right) + 220(1)\left(\frac{x^3}{64}\right)$$

$$= 1 + 3x + \frac{33}{8}x^2 + \frac{55}{16}x^3$$

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Question 3 continued

(b) $\left(\frac{x^2+8}{x^5}\right)\left(1+\frac{1}{4}x\right)^{12}$

$= \left(\frac{1}{x^3}+\frac{8}{x^5}\right)\left(1+\frac{1}{4}x\right)^{12}$

$= \left(\frac{1}{x^3}+\frac{8}{x^5}\right)\left(1+3x+\frac{33}{8}x^2+\frac{55}{16}x^3+Cx^4+Cx^5+\dots\right)$

For a value independant of x, where C is the coefficient of x.

$\frac{1}{x^3} \times Cx^3 = C$

$\frac{1}{\cancel{x^3}} \times \frac{55}{16}\cancel{x^3} = \frac{55}{16}$

$\frac{8}{x^5} \times Cx^5 = 8C$

where term with x^5 is : ${}^{12}C_5\, 1^{12-5}\left(\frac{x}{4}\right)^5$

$= \frac{12!}{(12-5)!\,5!}(1)\left(\frac{x^5}{1024}\right)$

$= 792(1)\left(\frac{x^5}{1024}\right) = \frac{99}{128}x^5$

$\frac{8}{\cancel{x^5}} \times \frac{99}{128}\cancel{x^5} = 8\left(\frac{99}{128}\right) = \frac{99}{16}$

term $= \frac{55}{16} + \frac{99}{16} = \frac{154}{16} = \frac{77}{8}$

Q3

(Total 6 marks)

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4. $f(x) = (x - 3)(3x^2 + x + a) - 35$ where a is a constant

(a) State the remainder when $f(x)$ is divided by $(x - 3)$.

(1)

Given $(3x - 2)$ is a factor of $f(x)$,

(b) show that $a = -17$

(2)

(c) Using algebra and showing each step of your working, fully factorise $f(x)$.

(5)

(a) REMAINDER THEOREM:

$f(x) = q(x)(x-c) + r$

remainder

quotient

divisor

SO:

this is our remainder!

$f(x) = (3x^2 + x + a)(x-3) + (-35)$

So remainder $= -35$

(b) Set the factor equal to zero:

$3x - 2 = 0 \longrightarrow x = \frac{2}{3}$

so; $f\left(\frac{2}{3}\right) = 0$

• substitute $x = 2/3$ into $f(x) = 0$:

$$f\left(\frac{2}{3}\right) = \left[3\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) + a\right]\left[\left(\frac{2}{3}\right) - 3\right] - 35 = 0$$

$$= (2 + a)\left(\frac{-7}{3}\right) - 35 = 0$$

$$-\frac{14}{3} - \frac{7a}{3} - 35 = 0$$

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Question 4 continued

$$-\frac{72}{\cancel{3}} = \frac{119}{\cancel{3}}$$

$$a = -17$$

(c) $f(x) = (x-3)(3x^2 + x - 17) - 35$

$$= 3x^3 + x^2 - 17x - 9x^2 - 3x + 51 - 35$$
$$= 3x^3 - 8x^2 - 20x + 16$$

Use algebraic division to find quotient :

$$3x - 2 \,\big)\, 3x^3 - 8x^2 - 20x + 16$$

(1) Divide first term of dividend by first term of divisor : $\frac{3x^3}{3x} = x^2 \rightarrow$ first term of quotient

$$\begin{array}{r|l} & x^2 \\ \hline 3x - 2 & 3x^3 - 8x^2 - 20x + 16 \end{array}$$

$\div$

(2) Multiply out and write under :

$$\begin{array}{r|l} & x^2 \\ \hline 3x - 2 & 3x^3 - 8x^2 - 20x + 16 \\ & 3x^3 - 2x^2 \\ \hline \end{array}$$

(3) Subtract then bring down next term :

$$\begin{array}{r|l} & x^2 \\ \hline 3x - 2 & 3x^3 - 8x^2 - 20x + 16 \\ & -(3x^3 - 2x^2) \\ \hline & -6x^2 - 20x \end{array}$$

(4) Divide first term of expression by first term of divisor : $\frac{-6x^2}{3x} = -2x \rightarrow$ next term of quotient

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Question 4 continued

$$
\begin{array}{r|l}
 & x^2 - 2x \\
\hline
3x-2 & 3x^3 - 8x^2 - 20x + 16 \\
 & -(3x^3 - 2x^2) \\
\hline
 & -6x^2 - 20x
\end{array}
$$

(5) Multiply out and write under:

$$
\begin{array}{r|l}
 & x^2 - 2x \\
\hline
3x-2 & 3x^3 - 8x^2 - 20x + 16 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 - 20x \\
 & -6x^2 + 4x \\
\hline
\end{array}
$$

(6) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & x^2 - 2x \\
\hline
3x-2 & 3x^3 - 8x^2 + 20x + 16 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 - 20x \\
 & -(-6x^2 + 4x) \\
\hline
 & -24x + 16
\end{array}
$$

(7) Divide first term of expression by first term of divisor: $\frac{-24x}{3x} = 8$ → next term of quotient

$$
\begin{array}{r|l}
 & x^2 - 2x + 8 \\
\hline
3x-2 & 3x^3 - 8x^2 - 20x + 16 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 - 20x \\
 & -(-6x^2 + 4x) \\
\hline
 & -24x + 16
\end{array}
$$

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Question 4 continued

(8) Multiply out and write under:

$$
\begin{array}{r|l}
 & x^2 - 2x + 8 \\
\hline
3x - 2 & 3x^3 - 8x^2 - 20x + 16 \\
 & (3x^3 + 2x^2) \\
\hline
 & -6x^2 - 20x \\
 & -(-6x^2 + 4x) \\
\hline
 & -24x + 16 \\
 & -24x - 16 \\
\hline
\end{array}
$$

(9) Subtract

$$
\begin{array}{r|l}
 & x^2 - 2x + 8 \\
\hline
3x - 2 & 3x^3 - 8x^2 - 20x + 16 \\
 & (3x^3 + 2x^2) \\
\hline
 & -6x^2 - 20x \\
 & -(-6x^2 + 4x) \\
\hline
 & -24x + 16 \\
 & -(-24x - 16) \\
\hline
 & 0
\end{array}
$$

can be factorised further!

so : $f(x) = (3x-2)(x^2 - 2x - 8)$

$= (3x-2)(x+2)(x-4)$

Q4

(Total 8 marks)

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5. (a) Given $0 < a < 1$, sketch the curve with equation

$$y = a^x$$

showing the coordinates of the point at which the curve crosses the y-axis.

(2)

Interval btwn two x-coords = h of trapezium

=0.5

x	2	2.5	3	3.5	4
y	4.25	6.427	9.125	12.34	16.06

y_1 y_5

multiplied by 2 as all are shared by two trapezia

The table above shows corresponding values of x and y for $y = x^2 + \left(\frac{1}{2}\right)^x$

The values of y are given to 4 significant figures as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x\right) dx$

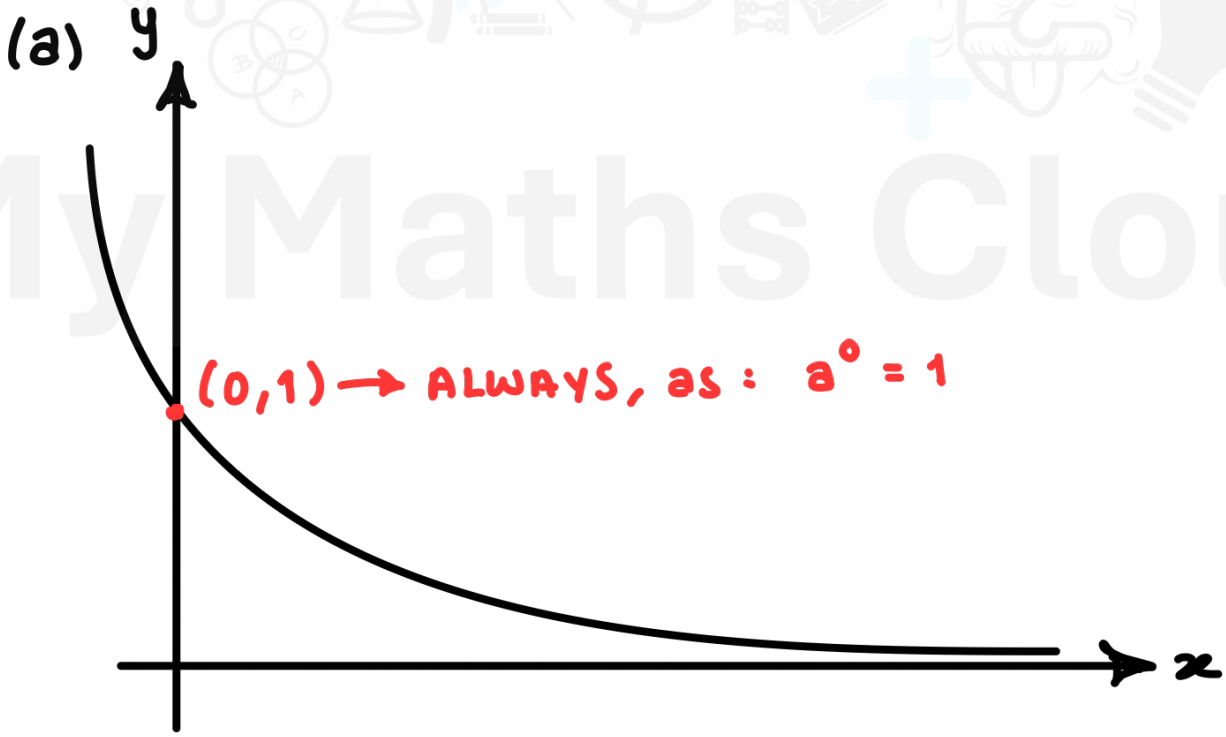
(3)

Using your answer to part (b) and making your method clear, estimate

(c) $\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x\right) dx$

(2)

(a)



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Question 5 continued

(b) (Formula is given in formula booklet.)

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4) + y_5\right]$$

$$= \frac{1}{2}(0.5)\left(4.25 + 2(6.427 + 9.125 + 12.34) + 16.06\right)$$

$$= 0.25(76.094) = 19.0235$$

$$= 19.0 \quad (3\text{ sf})$$

(c) $\int_2^4 \left[x(x-3) + \left(\frac{1}{2}\right)^x\right] dx$

$$\int_2^4 \left[x^2 - 3x + \left(\frac{1}{2}\right)^x\right] dx$$

$$= \int_2^4 \left[x^2 + \left(\frac{1}{2}\right)^x - 3x \; dx\right]$$

use rule : $\int f(x) \pm g(x)\, dx = \int f(x)dx \pm \int g(x)\, dx$

$$= \int_2^4 x^2 + \left(\frac{1}{2}\right)^x dx - \int_2^4 3x\, dx$$

REMEMBER!

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$= 19.0 - \left[\frac{3}{2}x^2\right]_2^4$$

$$= 19.0 - \left[\frac{3}{2}(4)^2 - \frac{3}{2}(2)^2\right]$$

$$= 19.0 - (24 - 6)$$

$$= 19.0 - 18 = 1.0$$

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Question 5 continued

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Question 5 continued

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6.

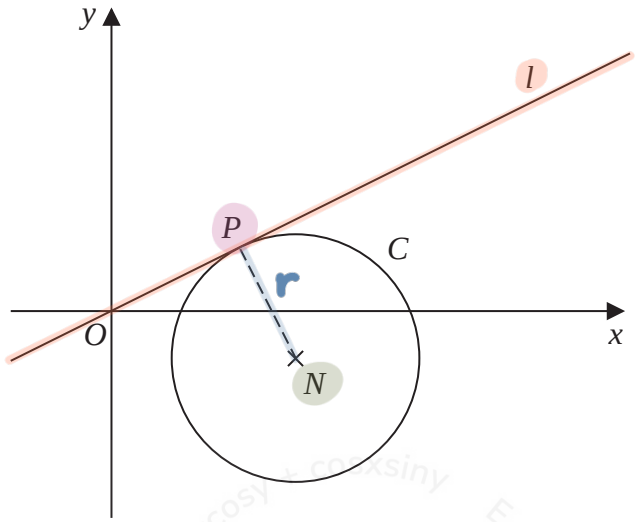

Figure 1

Figure 1 shows a sketch of a circle C with centre $N(4, -1)$.

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) the equation of C.

(5)

(a) PN is a radius. Since line l is a tangent, l and PN are perpendicular.

so : $m_{PN} = -\frac{1}{m_l}$

$m_{PN} = -\frac{1}{1/2} = -2$

eq. PN → $y = mx + c$

Point N lies on PN and has known coordinates : $(4, -1)$

so :

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Question 6 continued

Method 1 : USE : $y - y_1 = m(x - x_1)$

$y - (-1) = -2(x - 4)$

$y + 1 = -2x + 8$

$y = -2x + 7$

OR :

Method 2 : PLUG $(4, -1)$ IN $y = mx + c$, FIND c

$y = mx + c$

$-1 = -2(4) + c$

$-1 = -8 + c$

$c = 7$

SO : $y = -2x + 7$

(b) Eq. of circle :

$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$

① Find x, y :

point P lies on PN and line l. SO:

at P : $y_l = y_{PN}$

$\frac{1}{2}x = -2x + 7$

$\frac{5}{2}x = 7$

$x = \frac{14}{5}$

$y = \frac{1}{2}\left(\frac{14}{5}\right) = \frac{7}{5}$

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Question 6 continued

② Find length r = length PN

distance formula : $d = \sqrt{(x_1 - x_2)^2 + (y_2 - y)^2}$

$(r)^2 = \left(\sqrt{(14/5 - 4)^2 + (7/5 - (-1))^2}\right)^2$

$r^2 = \frac{36}{5}$

So, equation is :

$(x - 4) + (x \overset{+}{-} (-1)) = \frac{36}{5}$

$(x - 4) + (x + 1) = \frac{36}{5}$

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Question 6 continued

Q6

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7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\dfrac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\dfrac{\log_a a^2 b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

end terms :

sum starting value

Recall rules of logarithms ;

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$\log_a b^b = b\log_a a$

$\log_x x = 1$

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right) = \log_a\sqrt{a} - \log_a b$

$= \log_a a^{1/2} - \log_a b$

$= \frac{1}{2}\log_a a - \log_a b$

$= \frac{1}{2}(1) - \log_a b$

$= \frac{1}{2} - k$

(ii) $\dfrac{\log_a a^2 \times b}{\log_a b^3} = \dfrac{\log_a a^2 + \log_a b}{\log_a b^3}$

$= \dfrac{2\log_a a + \log_a b}{3\log_a b}$

$= \dfrac{2(1) + k}{k} = \dfrac{2+k}{k}$

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Question 7 continued

(iii)

$$\sum_{n=1}^{50} (k + \log_a b^n)$$

$$= \sum_{n=1}^{50} (k + n \log_a b)$$

$$= \sum_{n=1}^{50} (k + nk)$$

means start the sequence at $n=1$ and sum all the terms up to $n=50$

Use partial sum formula:

$$S_n = \frac{n}{2}\left[a_1 + a_n\right]$$

$$S_{50} = \frac{50}{2}\left((k+1k)+(k+50k)\right)$$

$$= 25(1k + 51k)$$

$$= 25(53k)$$

$$= 1325k$$

Q7

(Total 7 marks)

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8. Solutions relying on calculator technology are not acceptable in this question.

(i)

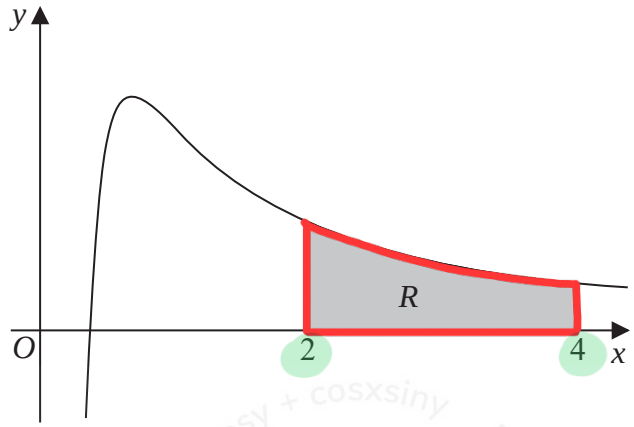

Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k \right) dx = 55$$

(4)

(i) ① Rewrite eq. for integration :

$y = \frac{8\sqrt{x} - 5}{2x^2}$ → $\sqrt{x} = x^{1/2}$

$y = \frac{8x^{1/2} - 5}{2x^2}$ → $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$y = \frac{8x^{1/2}}{2x^2} - \frac{5}{2x^2}$ → $\frac{x^a}{x^b} = x^{a-b}$

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Question 8 continued

$$y = 4x^{1/2-2} - \frac{5}{2x^2} \rightarrow \frac{1}{x^n} = x^{-n}$$

$$y = 4x^{-3/2} - \frac{5}{2}x^{-2}$$

② Integrate :

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$$\int_2^4 4x^{-3/2} - \frac{5}{2}x^{-2}\,dx$$

$$= \left[\frac{4x^{-3/2+1}}{-3/2+1} - \frac{5/2\,x^{-2+1}}{-2+1}\right]_2^4$$

$$= \left[-8x^{-1/2} + \frac{5x^{-1}}{2}\right]_2^4$$

$$= \left[-8(4)^{-1/2} + \frac{5}{2}(4)^{-1}\right] - \left[-8(2)^{-1/2} + \frac{5}{2}(2)^{-1}\right]$$

$$= \left(-4 + \frac{5}{8}\right) - \left(-4\sqrt{2} + \frac{5}{4}\right)$$

$$= 4\sqrt{2} - \frac{37}{8} \quad \text{(exact !)}$$

(ii) $$\int_{-3}^{6} \frac{1}{2}x^2 + k\,dx = 55$$

$$= \left[\frac{1/2\,x^{2+1}}{2+1} + \frac{kx^{0+1}}{0+1} + c\right]_{-3}^{6} = 55$$

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Question 8 continued

$$\left[\frac{1}{6}x^3 + kx\right]_{-3}^{6} = 55$$

$$\left(\frac{1}{6}(6)^3 + k(6)\right) - \left(\frac{1}{6}(-3)^3 + k(-3)\right) = 55$$

$$36 + 6k + \frac{9}{2} + 3k = 55$$

$$\frac{81}{2} + 9k = 55$$

$$9k = \frac{29}{2}$$

$$k = \frac{29}{18}$$

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Question 8 continued

Q8

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9. Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leqslant \theta < 180°$, the equation → calculator is in degrees

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$, → SWITCH CALCULATOR TO RADIANS!

(b) find, showing all working, the value of α to 3 decimal places.

(5)

(i) $3\sin(2\theta - 10°) = 1$, $2(0) - 10 \leq 2\theta - 10 < 2(180) - 10°$

let $(2\theta - 10°) = x$

$3\sin(2\theta - 10°) = 1$

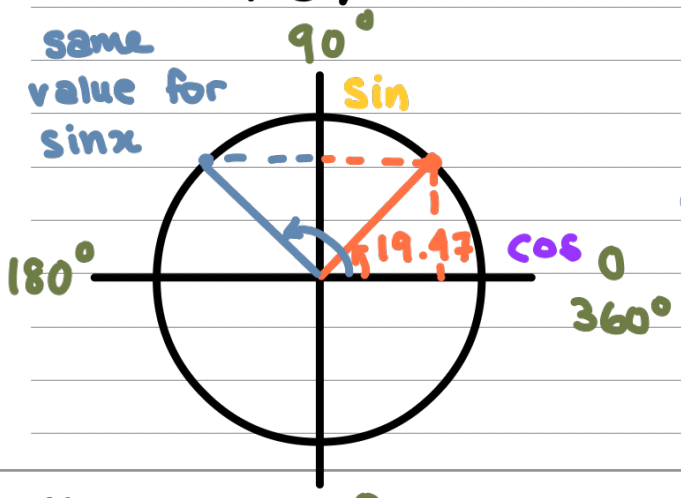
$3\sin x = 1$

$\sin x = \frac{1}{3}$, $0 \leq x < 180°$

USING CALCULATOR:

$x = \sin^{-1}\left(\frac{1}{3}\right) = 19.471°$

same value for sinx



ON UNIT CIRCLE:

$\sin\frac{1}{3} = 19.471°$

$\sin\frac{1}{3} = 180 - 19.471°$

$x = 160.528°$

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Question 9 continued

$x = 19.47^\circ, 160.53^\circ$

$2\theta - 10^\circ = 19.47^\circ, 160.53^\circ$

$+10^\circ \quad +10^\circ \quad +10^\circ$

$\frac{2\theta}{2} = \frac{29.47^\circ}{2}, \frac{170.53^\circ}{2}$

$\theta = 14.7^\circ, 85.3^\circ$ (1 d.p.)

(ii)(a) $a_1 \xrightarrow{+d} a_2 \xrightarrow{+d} a_3$

$\sin\alpha, \frac{1}{\tan\alpha}, 2\sin\alpha$

$d = a_2 - a_1$ AND $d = a_3 - a_2$

$d = \frac{1}{\tan\alpha} - \sin\alpha$ $\quad$ $d = 2\sin\alpha - \frac{1}{\tan\alpha}$

$d = d$

$\frac{1}{\tan\alpha} - \sin\alpha = 2\sin\alpha - \frac{1}{\tan\alpha}$

$\frac{2}{\tan\alpha} = 3\sin\alpha \rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{2}{\frac{\sin\alpha}{\cos\alpha}} = 3\sin\alpha$

$\sin\alpha \times \left(\frac{2\cos\alpha}{\sin\alpha}\right) = (3\sin\alpha) \times \sin\alpha$

$\therefore 2\cos\alpha = 3\sin^2\alpha$

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Question 9 continued

(b) $2\cos\alpha = 3\sin^2\alpha$

Use identity : $\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

$2\cos\alpha = 3(1 - \cos^2\alpha)$

$2\cos\alpha = 3 - 3\cos^2\alpha$

$3\cos^2\alpha + 2\cos\alpha - 3 = 0 \rightarrow$ FACTORISE

Let $\cos\alpha = x$

$3x^2 + 2x - 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(3)(-3)}}{2(3)}$

$x = \dfrac{-2 + \sqrt{40}}{6}$ or $\dfrac{-2 - \sqrt{40}}{6}$

So :

$\cos\alpha = \dfrac{-2 + \sqrt{40}}{6}$ $\left[\cos\alpha = \dfrac{-2 - \sqrt{40}}{6}\right]$

REJECT !

as $\cos\alpha = -1.387\ldots$ is not possible ; range of $y = \cos\alpha$ is $-1 < \alpha < 1$

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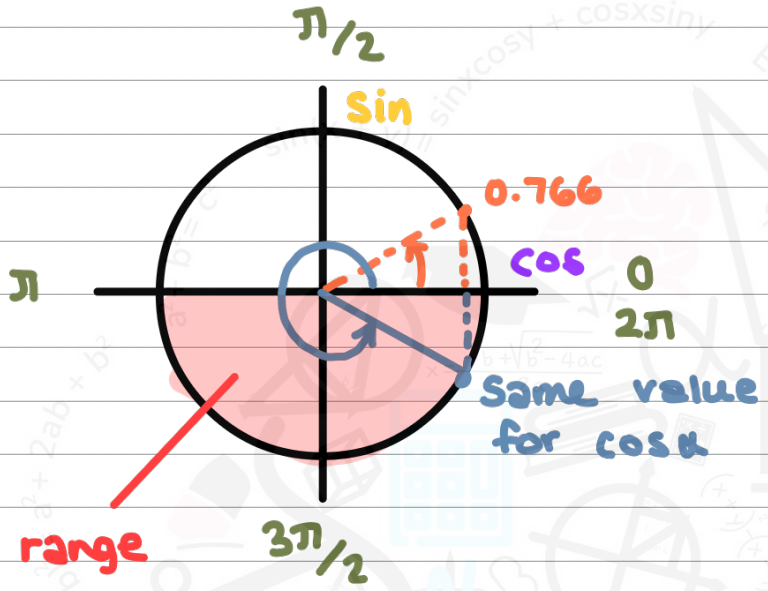
Question 9 continued

$$\cos\alpha = \frac{-2+\sqrt{40}}{6}$$

USE CALCULATOR :

$$\alpha = \cos^{-1}\left(\frac{-2+\sqrt{40}}{6}\right) = 0.766$$

ON UNIT CIRCLE :



$\alpha = 0.766 \rightarrow$ out of range

$\alpha = 2\pi - 0.766 = 5.517$

So: $\alpha = 5.517$ radians (3 d.p.)

Q9

(Total 12 marks)

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10. The curve C has equation

$$y = ax^3 - 3x^2 + 3x + b$$

where a and b are constants.

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7 $\quad \frac{dy}{dx} = 7$

(a) find the value of a and the value of b.

(4)

(b) Prove that C has no turning points.

(3)

(a) $y = ax^3 - 3x^2 + 3x + b$

① substitute $x, y = 2, 5$ to find eq #1 :

$5 = a(2)^3 - 3(2)^2 + 3(2) + b$

$5 = 8a - 12 + 6 + b$

$5 = 8a - 6 + b$

$8a + b = 11$

② Find dy/dx :

$y = ax^3 - 3x^2 + 3x + b$

$\frac{dy}{dx} = 3ax^{3-1} - (2)3x^{2-1} + (1)3x^{1-1}$

$= 3ax^2 - 6x + 3$

DIFFERENTIATION

$$\boxed{y = ax^n \qquad \frac{dy}{dx} = nax^{n-1}}$$

at $x = 2$, $dy/dx = 7$

$3a(2)^2 - 6(2) + 3 = 7$

$12a - 12 + 3 = 7$

$12a = 16$

$a = 4/3$

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Question 10 continued

③ Find b :

$8a + b = 11$

$8\left(\frac{4}{3}\right) + b = 11$

$b = \frac{1}{3}$

(b) At a turning point, $dy/dx = 0$

$y = \frac{4}{3}x^3 - 3x^2 + 3x + \frac{1}{3}$

$\frac{dy}{dx} = (3)\frac{4}{3}x^2 - (2)3x + 3$

$= 4x^2 - 6x + 3$

$4x^2 - 6x + 3 = 0$

$4x^2 - 6x + 3 = 0$

REMEMBER :

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ discriminant

If discriminant >0, then eq. has two solutions
If discriminant $=0$, then eq. has one solution
If discriminant <0, then eq. has no solutions

$b^2 - 4ac$
$= (-6)^2 - 4(4)(3)$
$= -12 < 0$ so NO ROOTS.

Since $dy/dx = 0$ has no solution, then there are no turning points ☹

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Question 10 continued

Q10

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

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